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Probabilistic Description of Traffic Breakdowns Caused by On-ramp Flow

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Abstract. The characteristic features of traffic breakdown near on-ramp are analyzed. To describe this phenomenon the probabilistic description regarding the jam emergence as the formation of a large car cluster on highway inside the synchronized traffic is constructed. In these terms the breakdown occurs through the formation of a certain critical nucleus in the metastable vehicle flow, which is located near the on-ramp. The strong cooperative car interaction in the synchronized traffic enables us to treat the size of critical jam nuclei as a large value and to apply to an effective one-lane model. This model assumes the following. First, the growth of a car cluster is governed by the attachment of cars to the cluster whose rate is mainly specified by the total traffic flow. Second, the cluster dissolution is determined by the car escape from the cluster whose rate depends on the cluster size directly. Third, the generation of one-car clusters (preclusters) is caused by cars entering the main road from the on-ramp. The appropriate master equation for the car cluster evolution is written and the generation rate of critical jam nuclei is found. The obtained results are in agreement with the empirical facts that the characteristic time scale of the breakdown phenomenon is about or greater than one minute and the traffic flow rate interval inside which traffic breakdowns are observed is sufficiently wide. Besides, as a new results, it is shown that the traffic breakdown probability can be analyzed, at least approximately, based solely on the data of the total vehicle flow without separating it into the vehicle streams on the main road and on-ramp when the relative on-ramp flow volume exceeds 10%–20%.

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1 Introduction

For the last decade physics of traffic flow held attention of physical society due to two reasons. The former is its obvious importance for traffic engineering especially concerning the feasibility of attaining the limit capacities of traffic networks and quantifying it. The latter is related to the fact that vehicle ensembles on highways form a sufficiently simple example of systems with motivation being the object of new branches in modern physics. Indeed, on one hand, the individual motion of cars is affected essentially by the driver behavior in addition to the regularities of classical mechanics. So, in this sense, the vehicle ensembles are nonphysical systems. On the other hand, on the macroscopic level the vehicle ensembles exhibit a lot of properties like phase formation and phase transitions widely met in physical systems (for a review see [1,2,3]).

The traffic breakdown, i.e. the initial stage of jam formation typically near bottlenecks is an important phenomenon for traffic engineering, exactly its main characteristics determine the limit capacity of the corresponding road fragments or nodes. Its properties are sufficiently complex, traffic breakdown usually proceeds through the sequence of two phase transitions: free flow → synchronized traffic → stop-and-go pattern with a number of hysteresis and nucleation effects [4,5]. A detailed description of the jam formation near bottlenecks can be found in Ref [6]. Nevertheless the basic properties of traffic breakdown are far from being understood well. In particular, it is a probabilistic phenomenon [7,8], i.e. traffic breakdown occurs not immediately after the vehicle flow rate attaining a certain critical value but randomly within some interval (q_{c1}, q_{c2}). It is easy to explain this assuming homogeneous traffic flow to be metastable and a jam to emerge via the nucleation mechanism. A nontrivial fact is a large width of the traffic breakdown interval. According to the empirical data [8,9] $q_{c1} \sim$ 1000–1500 v/h/l (vehicle per hour per lane) and $q_{c2} \sim$ 1900–2800 v/h/l, i.e. the ratio

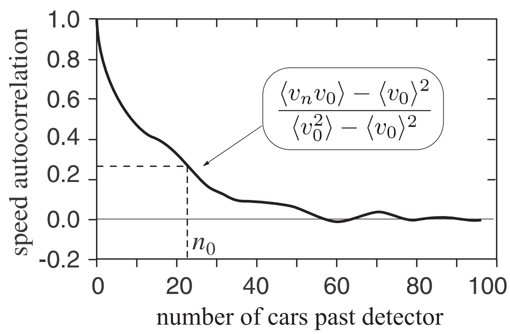

Fig. 1. Illustration of the speed autocorrelation *vs* the number of cars that have passed a fixed detector. Based on the observations by Neubert, Santen, Schadschneider, and Schreckenberg [12].

$(q_{c2} - q_{c1})/q_{c1} \sim 50\%–100\%$. The traffic breakdown phenomenon is also characterized by a large time interval during which it develops. Namely, it is about several minutes, a typical observation time of detecting traffic breakdown according to the traffic engineering technique. By contrast, the characteristic time scale of individual car dynamics is about or less than ten seconds.

In the previous paper [11] we have developed a simple model explaining the two last features of traffic breakdown. Its basic point is the assumption that the synchronized phase of traffic flow develops first and traffic breakdown, in its own accord, comes into being via the formation of a critical jam nucleus inside this phase with strong cooperative interaction between cars. This corresponds to the recent notion about the characteristic properties of traffic flow near bottlenecks [4,5,6]. The size n_0 of such critical nuclei must be sufficiently large, $n_0 \sim 10–20$, which is justified by the analysis of single vehicle data [12] and illustrated in Fig. 1. As seen in Fig. 1 a car cluster in the synchronized traffic must span over many cars along the lane. Such a car cluster also can span over all the lane on a highway, which enable us to apply to an effective one-lane approximation dealing with macrovehicles rather than real individual cars.

This model, however, has left the question about the jam origin or, what is the same, about the source of one-car clusters (preclusters) beyond the consideration. Typically traffic breakdowns are caused by the influence of different bottlenecks, in particular, on- or off-ramps. Exactly vehicles entering or leaving traffic flow on the main road are source of jam nuclei. Investigation of the effect of the on-ramp flow on the traffic breakdown is the subject of the present work. As a particular result we would like to find an justification for the analysis of traffic breakdown phenomena near on-ramps using only the data of the total traffic flow leaving the on-ramp region. This is a typical way of analyzing real empirical data because of the lack of statistics.

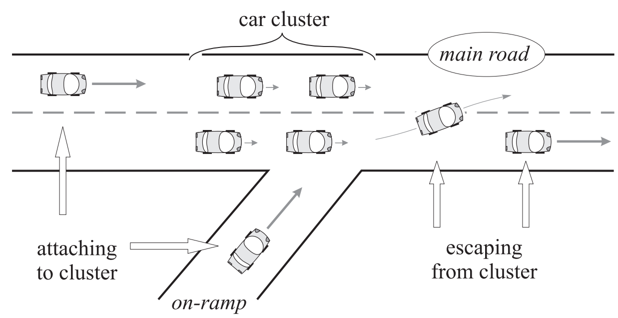

Fig. 2. The mechanism under consideration that governs the formation and growth of a car cluster on a highway near an on-ramp

2 Model: Car cluster growth near on-ramp

The proposed model for the formation of a car cluster caused by on-ramp vehicle flow and its further growth is illustrated in Fig. 2. The model is based on the following assumptions. First, each car entering the highway from the on-ramp either attaches itself to an existing car cluster or forms a precluster (one-car cluster) which further can grow or dissipate. Second, when a car moving on the highway reaches the cluster it attaches itself to the cluster. Third, due to the lane change maneuvers the escape rate of cars from the cluster depends on the cluster size. Fourth, the formed car cluster is characterized by a small mean velocity so it is practically localized near the on-ramp.

The given model takes into account that a real jam nucleus develops inside the synchronized traffic on a multilane highway and it exhibits irreversible growth only after its size n attaining a sufficiently large critical value $n_c \gg 1$. Due to the strong multilane cooperative effects in the synchronized traffic all the vehicles interact with suhc a cluster. For a small jam nucleus real cars either can avoid it by changing the lanes or leave the cluster fast because its mean velocity has no time to drop sufficiently low. This is formally allowed for by the dependence of the escape rate on the cluster size n. So the proposed effective one-lane model imitates a more complex phenomenon of traffic breakdown on highways.

The adopted assumptions lead to the scheme of the car cluster dynamics shown in Fig. 3. Each car attachment to or detachment from the cluster causes unit change of its size whose sequence is considered to be mutually independent. The attachment rate to the cluster of size $n \geq 1$ is written in the form

$$w_+ = q_{\text{in}} + q_{\text{ramp}} \,, \tag{1}$$

where q_{in} is the rate of vehicle flow on the main road entering the on-ramp region (per lane) and q_{ramp} is the rate of traffic flow on the on-ramp. The rate of one-cluster generation (precluster generation) is

$$w_+^0 = q_{\text{ramp}} := \epsilon w_+ \,. \tag{2}$$

Here the latter equality is no more than the definition of the value ϵ treated as a small parameter, $\epsilon \ll 1$. Expression (2) is the mathematical implementation of the

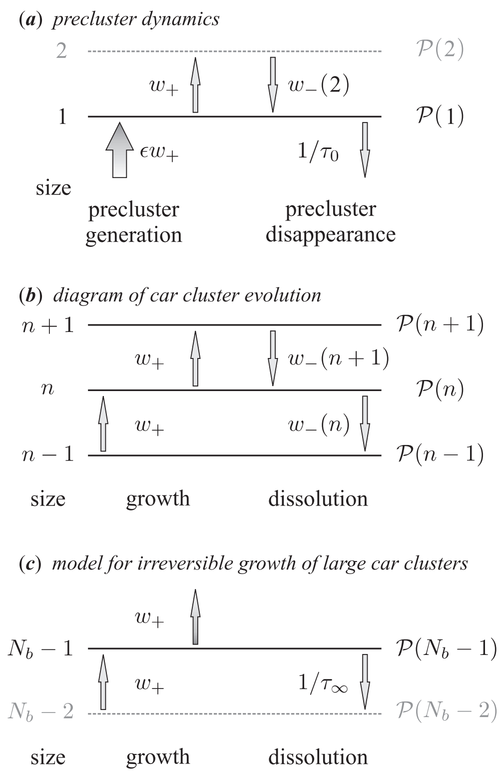

Fig. 3. Schematic illustration of the cluster transformations.

assumptions that the car clusters are initially generated solely by cars from the on-ramp and, thus, without the on-ramp vehicle flow no traffic breakdown is possible. Finally, the detachment rate from the car cluster of size n is specified by the *Ansatz*

$$w_-(n) = \frac{1}{\tau(n)} := \frac{1-\phi(n)}{\tau_\infty} + \frac{\phi(n)}{\tau_0}\,, \tag{3}$$

where the function $\phi(n)$ decreases from 1 to 0 as the cluster size n runs from 1 to ∞. Time scales $\tau_0 < \tau_\infty$ characterize the detachment rate from small clusters and sufficiently large ones, respectively. Their values can be estimated from the relations $q_{c1} \approx 1/\tau_\infty$ and $q_{c2} \approx 1/\tau_0$, where q_{c1} and q_{c2} are the traffic flow rates (per lane) such that a jam cannot form at all for $q < q_{c1}$ and a jam emerges immediately when the flow rate q exceeds q_{c2} (see Section 1). In these estimates we do not distinguish between the flow rate on the main road $q_{\rm in}$ and the total flow rate $q_{\rm in} + q_{\rm ramp}$ because typically $q_{\rm ramp} \ll q_{\rm in}$. Applying to the available empirical data [8,9] we set, for example, $q_{c1} \sim 1500$ v/h/l and $q_{c2} \sim 2500$ v/h/l whence get $\tau_0 \sim 1.5$ sec and $\tau_\infty \sim 2.5$ sec. Besides, as also discussed in Section 1 a scale n_0 dividing the car clusters into small and large ones is much greater than unity, $n_0 \sim 10$–20, due to the cooperative interaction of cars in the synchronized traffic. The form of the $w_-(n)$-dependence is schematically shown in Fig. 4.

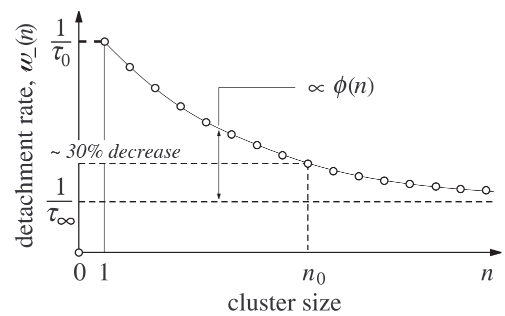

Fig. 4. The detachment rate $w_-(n)$ *vs* the cluster size n. A qualitative sketch.

To be specific in obtaining some numerical estimates the following simple *Ansatz*

$$\phi(n) = \frac{n_0+1}{n_0+n}\,, \tag{4}$$

will be used, however, the particular form of the $\phi(n)$-dependence is of minor effect.

The car cluster evolution is described by the dynamics of the distribution function $\mathcal{P}(n,t)$ which, by virtue of Diagram 3, obeys the forward master equation (following Mahnke *et al.* [13,14])

$$\begin{aligned}\partial_t \mathcal{P}(n,t) = w_+ \mathcal{P}(n-1,t) + w_-(n+1)\mathcal{P}(n+1,t)\\ - [w_+ + w_-(n)]\mathcal{P}(n,t)\end{aligned} \tag{5}$$

for $n \geq 2$. The precluster evolution is described by the equation

$$\partial_t \mathcal{P}(1,t) = w_+^0 - [w_+ + w_-(1)]\mathcal{P}(1,t)\,. \tag{6}$$

The model under consideration describes only the initial stage of jam emergence, i.e. the formation of the jam critical nucleus. When the size n of a car cluster exceeds a certain critical value n_c it undergoes the irreversible growth giving rise to the jam formation. Within the frameworks of the adopted description this effect is taken into account by the following “boundary” condition imposed on the distribution function $\mathcal{P}(n,t)$ taken at a sufficiently distant point $N_b \gg n_0$:

$$\mathcal{P}(N_b,t) = 0\,. \tag{7}$$

Naturally, in this case equation (5) holds at points $2 \leq n \leq N_b - 1$. The system of equations (5)–(7) makes up the proposed model.

In what follows the assumption

$$\frac{1}{\tau_\infty} \approx q_{c1} < q_{\rm in} < q_{c2} \approx \frac{1}{\tau_0} \tag{8}$$

will be adopted. In other words, we will confine our consideration to the case when the traffic breakdown can occur but the homogeneous state of the vehicle flow is locally stable. Exactly in this case the traffic breakdown exhibits the probabilistic behavior. Then assuming the traffic flow rate on the on-ramp and the main road to be fixed the

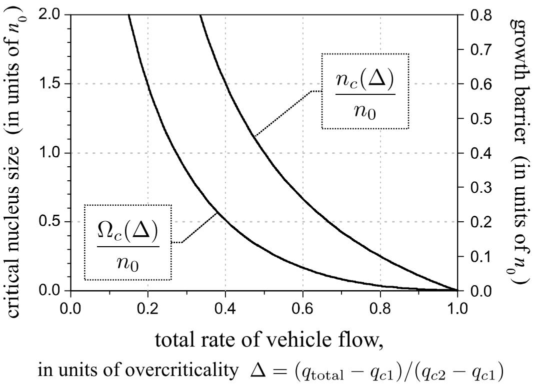

Fig. 5. The size of critical jam nuclei, n_c, and the growth barrier Ω_c vs the total traffic flow rate $q_{\rm total} = q_{\rm in} + q_{\rm ramp}$ measured in the units of the overcriticality $\Delta = (q_{\rm total} - q_{c1})/(q_{c2} - q_{c1})$. In obtaining these curves we used *Ansatz* (4) and set $(q_{c2} - q_{c1})/q_{c1} = 1.0$.

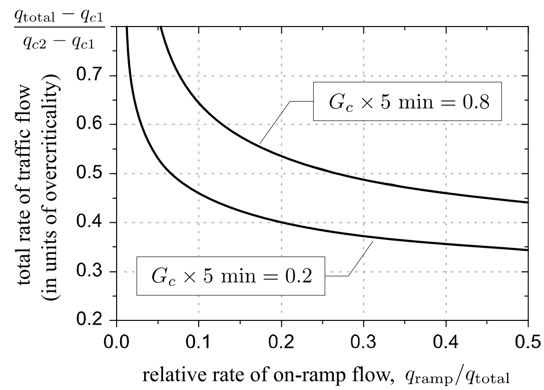

Fig. 6. Solid curves on the phase plane "on-ramp flow rate – total vehicle flow rate" depict the loci where the generation rate G_c of jam critical nuclei takes the values 0.2/5 min and 0.8/5 min, respectively. Roughly speaking, these curves correspond to points on this phase plane where the probability of traffic breakdown within 5-min observation interval is about 20% and 80%, respectively. In obtaining these curves we used *Ansatz* (4) and set $q_{c1} = 1500$ veh/h/l, $q_{c2} = 3000$ veh/h/l, and $n_0 = 10$.

given model enables us to calculate directly the generation rate of jam critical nuclei G_c, what is done in Appendix A. Using the obtained formula (24) and the relation (2) the desired value G_c can be written as

$$G_c \approx q_{\rm ramp} \sqrt{\frac{\beta_c}{2\pi n_c}} \exp\left\{-\Omega(n_c)\right\} . \tag{9}$$

Here the critical car cluster size n_c is specified by equation (21), $\Omega(n_c)$ is the "growth" potential (20) taken at the point $n = n_c$, and the constant $\beta_c \sim 1$ is determined by expansion (22). The car clusters have to overcome exactly the potential barrier $\Omega(n_c)$ for their growth to become irreversible.

Figure 5 depicts the dependence of the critical cluster size $n_c(\Delta)$ as well as the growth barrier $\Omega(n_c) := \Omega_c(\Delta)$ on the total traffic flow rate $q_{\rm total} := q_{\rm in} + q_{\rm ramp}$ in units of the overcriticality

$$\Delta := \frac{q_{\rm total} - q_{c1}}{q_{c2} - q_{c1}} \tag{10}$$

that were obtained using *Ansatz* (4). The corresponding form of the critical cluster generation rate G_c is (see Appendix A, expression (28))

$$G_c = \frac{q_{\rm ramp}}{\sqrt{2\pi n_0}} \sqrt{\frac{q_{c2} - q_{c1}}{q_{\rm total}}} \left(\frac{q_{c2}}{q_{\rm total}}\right)^{\frac{n_0 q_{c2}}{q_{c1}}} \Delta^{1+\frac{n_0(q_{c2}-q_{c1})}{q_{c1}}} . \tag{11}$$

Figure 6 exhibits the characteristic features of the obtained critical cluster generation rate depending on the volumes of the on-ramp flow and the total vehicle flow.

3 Conclusion

The present paper developed a probabilistic description of the traffic breakdown phenomena near an on-ramp. Previously [11] we have proposed a simple probabilistic model for the traffic breakdown that explains two facts observed empirically (see Refs [8,9] as well for a review Ref. [3]). The former is the large width of the traffic flow rate interval (q_{c1}, q_{c2}) wherein the traffic breakdown phenomena are observed, namely, $(q_{c2} - q_{c1})/q_{c1} \sim$ 50%–100%. The latter is the fact that the time interval during which traffic breakdown develops is about several minutes whereas the characteristic time scale of the individual car dynamics is about or less than ten seconds. However the origin of traffic breakdown has been left beyond the analysis. Typically different kinds of bottlenecks, for example, on- and off-ramps, are responsible for the jam emergence. The main purpose of the given paper was to take into account directly the effect of on-ramp within this probabilistic description. Besides we would like to explain why the empirical analysis tackling the traffic breakdown near on-ramps can be performed dealing solely with the data of the total traffic flow without separating it into main road and on-ramp streams. It is a typical situation because of the lack of statistics.

According to the modern notion of jam emergence it proceeds mainly through the sequence of two phase transitions: free flow → synchronized mode → stop-and-go pattern [5]. Both of these transitions are of the first order, i.e. they exhibit breakdown, hysteresis, and nucleation effects [4]. However, in the jam formation the second transition typically plays the leading role (for a detailed analysis see Ref. [6]).

Therefore the proposed model assumes, as the basic point, that a jam develops inside a certain mode of traffic flow with strong cooperative car interaction. In other words, cars entering the main road from the on-ramp give rise to a cooperative phase of car motion. Exactly inside this phase jam nuclei occur and lead to the irreversible jam formation when their size exceeds randomly a certain

critical value. Due to the cooperative car interaction this critical size must be much larger than unity, as it follows from the available single vehicle data [10]. Besides, in this case a critical jam nucleus has to span over all the lane, which enabled us to use an effective one-lane approximation actually dealing with macrovehicles rather than real cars.

The effect of complex interaction between the cars moving on the main road and entering the neighborhood of the on-ramp with a jam nucleus located near the on-ramp is taken into accounts as follows. A jam nucleus is treated as a cluster of car moving sufficiently slow near the on-ramp. Each car entering the on-ramp neighborhood with a car cluster attaches itself to it. In this approach the rate of the cluster growth w_+ is considered to be determined completely by the vehicle flow on the main road entering the on-ramp region as well as by the traffic flow on the on-ramp. The car detachment process is described by the escaping rate $w_-(n)$ depending on the cluster size n and decreasing with n. Therefore both the facts that cars can avoid a small jam nucleus by changing the lanes and overtaking it as well as can escape it also changing the lanes are allowed for by the $w_-(n)$-dependence.

What is new in the model under consideration in comparison with the previous one [11] is the assumption that one-car clusters, i.e. car preclusters being the initial state of jam nuclei are due to cars entering the main road from the on-ramp. Expression (9) or its particular form (11) specifies the desired generation rate G_c of the critical jam nuclei depending, in particular, on the on-ramp flow rate. The obtained result is illustrated in Fig. (6).

The main conclusion of the present paper is the following. When the vehicle flow rate, q_{ramp}, exceeds 10%–20% of the total traffic flow rate, q_{total}, the characteristics of the traffic breakdown depend weakly on q_{ramp} individually. So in this case the traffic breakdown phenomenon can be analyzed, at least semiqualitatively, using solely the total flow rate data. Otherwise, $q_{\mathrm{ramp}} \lesssim q_{\mathrm{total}}$, the details of partitioning the traffic flow rate into the main and on-ramp streams are the factor. In addition, the given model, as previous one [11], explains the large width of the vehicle flow rate interval (q_{c1}, q_{c2}) wherein traffic breakdowns are observed. It relates the critical values q_{c1}, q_{c2} to the dependence of the car detachment rate $w_-(n)$ on the cluster size n, namely, $q_{c1} = w_-(\infty)$ and $q_{c2} = w_-(1)$. Since a car cluster forms inside the synchronized traffic where the car cooperative interaction is strong the characteristic value n_0 separating car clusters into "small" and "large" is much greater than unity. Therefore the ratio $(w_1 - w_\infty)/w_\infty$ has to be about unity. Besides, as follows from expression (9) the characteristic time scale of the traffic breakdown development is about $\tau_{\mathrm{bd}} \sim \sqrt{2\pi n_0}(q_{c1}/q_{\mathrm{ramp}})\tau_\infty \gtrsim 1$ min for $q_{\mathrm{ramp}} \sim 0.2 q_{c1}$. This estimate explains ones more why the traffic breakdown phenomena are typical detected within 5–15 minute intervals.

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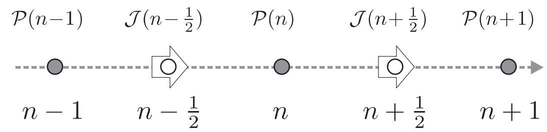

Fig. 7. Illustration of the car cluster "space" and the quantities defined in it. The solid circles depict the integers $\{n\}$, the cluster size values, at which the distribution function $\mathcal{P}(n,t)$ is defined. The hole circles match the intermediate points $\{n+\frac{1}{2}\}$ at which the cluster flux $\mathcal{J}(n+\frac{1}{2}, t)$ is considered.

A Steady-state flux of jam nuclei

The system of equations (5)–(7) describes the evolution of jam nuclei at the initial stage of jam emergence. In terms of the cluster flux $\mathcal{J}(n+\frac{1}{2}, t)$ defined in the car cluster "space" (Fig. 7)

$$\mathcal{J}(n+\tfrac{1}{2}, t) := w_+ \mathcal{P}(n,t) - w_-(n+1)\mathcal{P}(n+1,t) \quad (12)$$

this model can be reduced to the governing equation holding for $1 \le n \le N_b - 1$

$$\partial_t \mathcal{P}(n,t) = \mathcal{J}(n-\tfrac{1}{2}, t) - \mathcal{J}(n+\tfrac{1}{2}, t) \quad (13)$$

subject to the following two "boundary" conditions

$$\mathcal{J}(\tfrac{1}{2}, t) = \epsilon w_+ - w_-(1)\mathcal{P}(1,t), \quad (14)$$

$$\mathcal{J}(N_b - \tfrac{1}{2}, t) = w_+ \mathcal{P}(N_b - 1, t). \quad (15)$$

Here equation (13) is no more than equation (5) rewritten in the new terms, whereas values (14) and (15) are the precluster flux and the flux of large car cluster caused by their irreversible growth, respectively. The two expressions stem directly from equations (6), (7) and are illustrated in Fig. 3*a,c*.

The steady-state solution of system (13)–(15), in particular, the steady-state value of the cluster flux $G_c := \mathcal{J}(N_b - \frac{1}{2})$ gives us the desired generation rate of jam critical nuclei. Naturally, in this case the on-ramp flow rate as well as the traffic flow rate on the main road have to be treated as constant in time values.

The steady-state condition leads to a constant value of the car cluster flux $\mathcal{J}(n-\frac{1}{2}) = G_c$ which together with the "boundary" condition (15) gives us the relation

$$\mathcal{P}_s(n) = \left\{1 + \sum_{p=1}^{N_b-1-n} \prod_{q=1}^{p} \frac{w_-(n+q)}{w_+}\right\} \frac{G_c}{w_+}. \quad (16)$$

The substitution of (16) into (14) yields the desired expression for the generation rate of jam critical nuclei

$$G_c = \epsilon w_+ \left\{1 + \sum_{p=1}^{N_b-1} \exp\left[\sum_{q=1}^{p} \ln\left(\frac{w_-(q)}{w_+}\right)\right]\right\}^{-1}, \quad (17)$$

which is the main result of Appendix A.

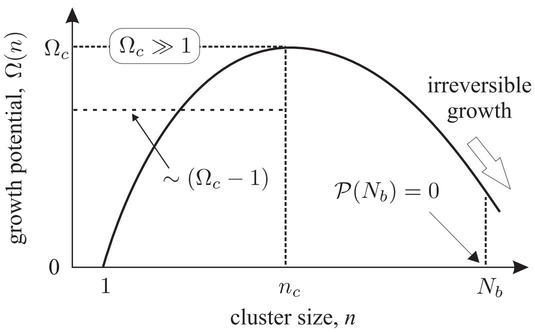

Fig. 8. Schematic illustration of the growth potential and the corresponding characteristic values of the cluster size.

Continuum approximation

When the inequalities

$$w_-(\infty) < w_+ < w_-(1)\,, \tag{18}$$

$$(w_+ - w_-(\infty))n_0\,,\ (w_-(1) - w_+)n_0 \gg 1 \tag{19}$$

hold expression (17) is simplified. Since the characteristic scale n_0 on that the quantity $w_-(n)$ exhibits substantial decrease is large, $n_0 \gg 1$, the sums in expression (17) can be replaced by the corresponding integrals. Indeed, let us introduce the "growth potential"

$$\Omega(n) := \sum_{q=1}^{n} \ln\left(\frac{w_-(q)}{w_+}\right). \tag{20}$$

Its form is illustrated in Fig. 8. In particular, the growth potential attains the maximum at the point n_c being the root of the equation

$$w_-(n_c) = w_+ \tag{21}$$

and playing the role of the critical size of jam nuclei.

In the vicinity of the point n_c the value $w_-(n)$ can be treated as a function of the continuous argument n and approximated by the expression

$$w_-(n) \approx w_+\left[1 - \beta_c \frac{(n - n_c)}{n_c}\right], \tag{22}$$

where $\beta_c \sim 1$ is a constant about unity. Since the main contribution to the first sum in expression (17) is due to a certain neighborhood of the point n_c we approximate the growth potential as follows

$$\Omega(n) = \Omega(n_c) - \beta_c \frac{(n - n_c)^2}{2n_c}\,. \tag{23}$$

Substituting formula (23) into expression (17) and replacing the sum running over p by the corresponding integral over the continuous variable p we get

$$G_c \approx \epsilon w_+ \sqrt{\frac{\beta_c}{2\pi n_c}} \exp\{-\Omega(n_c)\}\,. \tag{24}$$

Formula (24) is the desired expression for the generation rate of jam critical nuclei.

In particular, in the given limit for *Ansatz* (3) the critical size n_c of jam nuclei is specified by the expression

$$\phi(n_c) = \Delta\,, \tag{25}$$

where the traffic flow overcriticality measure Δ was introduced by formula (10) and we have set $q_{c1} = 1/\tau_\infty$ and $q_{c2} = 1/\tau_0$. Using in addition *Ansatz* (4) we get from formulae (20) and (22) the expressions for the parameter

$$\frac{\beta_c}{n_c} = \frac{(q_{c2} - q_{c1})}{q_{\text{total}}} \frac{\Delta^2}{n_0} \tag{26}$$

and for the critical potential barrier

$$\Omega(n_c) = -n_0 \left[\frac{(q_{c2} - q_{c1})}{q_{c1}} \ln \Delta + \frac{q_{c2}}{q_{c1}} \ln\left(\frac{q_{c2}}{q_{\text{total}}}\right)\right]\,. \tag{27}$$

Substitution of expressions (26) and (27) into (24) yields

$$G_c \approx \frac{\epsilon w_+}{\sqrt{2\pi n_0}} \sqrt{\frac{q_{c2} - q_{c1}}{q_{\text{total}}}} \left(\frac{q_{c2}}{q_{\text{total}}}\right)^{\frac{n_0 q_{c2}}{q_{c1}}} \Delta^{1 + \frac{n_0(q_{c2} - q_{c1})}{q_{c1}}}\,, \tag{28}$$

which is exactly formula (11) due to relation (2).

References

1. B.S. Kerner, in *Transportation and Traffic Theory* edited by A. Ceder (Pergamon, Amsterdam 1999), p. 147.
2. D. Chowdhury, L. Santen, and A. Schadschneider, Phys. Rep. **329** (2000), 199.
3. D. Helbing, Rev. Mod. Phys. **73** (2001), 1067.
4. B. S. Kerner and H. Rehborn, Phys. Rev. Lett. **79** (1997), 4030.
5. B. S. Kerner. Phys. Rev. Lett. **81** (1998), 3797.
6. B. S. Kerner and S. L. Klenov, Phys. Rev. E **68** (2002), 036130.
7. Elefteriadou, L., R.P. Roess, and W.R. McShane, Transp. Res. Rec. **1484**(1995), 80.
8. B. Persaud, S. Yagar, and R. Brownlee, Transp. Res. Rec. **1634** (1998), 64.
9. M. Lorenz and L. Elefteriadou, Transport. Res. Circular **E-C018**, 84 (2000).
10. L. Neubert, L. Santen, A. Schadschneider, and M. Schreckenberg. Phys. Rev. E **60** (1999), 6480.
11. R. Kühne, R. Mahnke, I. Lubashevsky, and J. Kaupužs, Phys. Rev. E **65** (2002), 066125.
12. L. Neubert, L. Santen, A. Schadschneider, and M. Schreckenberg, Phys. Rev. E **60** (1999), 6480.
13. R. Mahnke and N. Pieret, Phys. Rev. E **56** (1997), 2666.
14. R. Mahnke and J. Kaupužs, Phys. Rev. E **59** (1999), 117.